UNIVERSITY OF CALCUTTA

Notification No. CSR/ 64/18

It is notified for information of all concerned that the Syndicate in its meeting held on 13.07.2018 (vide Item No.11) approved the Syllabus and Regulations of Two-Year (Four-Semester) M.Sc. Course of Study in Applied Mathematics under CBCS in the Post-Graduate Departments of the University and in the affiliated Colleges offering Post-Graduate Courses under this University, as laid down in the accompanying pamphlet.

The above shall be effective from the academic session 2018-2019.

SENATE HOUSE
KOLKATA-700073
The 17th August, 2018

(Debabrata Marina)
Deputy Registrar (Acting)
DEPARTMENT OF APPLIED MATHEMATICS

UNIVERSITY OF CALCUTTA

M. Sc. (CBCC) SYLLABUS

WITH EFFECT FROM ACADEMIC SESSION 2018-19
REGULATIONS FOR TWO-YEAR (FOUR SEMESTER) M.Sc. DEGREE COURSE IN APPLIED MATHEMATICS UNDER THE UNIVERSITY OF CALCUTTA

1. The examination for the degree of Master of Science (M.Sc.) shall consist of four semesters, Semesters 1, 2, 3 and 4. Each semester examination will be held after the completion of the modules for that particular semester and before the next semester begins, the University authority will decide commencing on such date and time and it will be duly notified. The total duration of the course is two years (hereafter, “course” refers to the M.Sc. course in Applied Mathematics and “module” refers to the individual modules of 50 marks each). The duration of the semester examinations shall be as follows ordinarily:

<table>
<thead>
<tr>
<th>Semester</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Semester</td>
<td>July to December</td>
</tr>
<tr>
<td>2nd Semester</td>
<td>January to June</td>
</tr>
<tr>
<td>3rd Semester</td>
<td>July to December</td>
</tr>
<tr>
<td>4th Semester</td>
<td>January to June</td>
</tr>
</tbody>
</table>

The structure of the revised syllabus for the M.Sc. course in Applied Mathematics applicable from the academic year 2017-18, will be as follows:

**FIRST SEMESTER MODULES**

<table>
<thead>
<tr>
<th>Module Code</th>
<th>Module Name</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMATH 411</td>
<td>Classical Mechanics and Introductory ............</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Quantum Theory</td>
<td></td>
</tr>
<tr>
<td>AMATH 412</td>
<td>Gr-A: Abstract Algebra</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Gr-B: Linear Algebra</td>
<td>20</td>
</tr>
<tr>
<td>AMATH 413</td>
<td>Real Analysis</td>
<td>50</td>
</tr>
<tr>
<td>AMATH 414</td>
<td>Gr-A: Complex Analysis</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Gr-B: Theory of Ordinary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Differential Equations</td>
<td>20</td>
</tr>
<tr>
<td>AMATH 415</td>
<td>Gr-A: Partial Differential Equations</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Gr-B: Generalised Functions &amp; Integral Transforms</td>
<td>25</td>
</tr>
</tbody>
</table>
SECOND SEMESTER MODULES

AMATH 421  Gr-A: Continuum Mechanics I.................................25 Marks
            Gr-B: Rigid Dynamics..............................................25 Marks
AMATH 422  Continuum Mechanics II........................................50 Marks
AMATH 423  Topology, Functional Analysis & Operator..............50 Marks
AMATH 424  Gr-A: Optimization Techniques..............................30 Marks
            Gr-B: Calculus of Variations.................................20 Marks
AMATH 425  Gr-A: Theory of Relativity ........................................25 Marks
            Gr-B: Classical Electromagnetism..............................25 Marks

THIRD SEMESTER MODULES

AMATH 531  Gr-A: Discrete Mathematics, Graph Theory...........40 Marks
            Gr-B: Nonlinear Dynamics.....................................10 Marks
AMATH 532  Numerical Analysis..............................................50 Marks
AMATH 533  Advanced Paper I..................................................50 Marks
CBCC-A*   Choice Based Credit Course-A.................................50 Marks
CBCC-B*   Choice Based Credit Course-B.................................50 Marks

*See Annexure – I
*The courses are for the students of other disciplines.

FOURTH SEMESTER MODULES

AMATH 541  Numerical Practical ..............................................50 Marks
AMATH 542  Gr-A: Stochastic Process .......................................25 Marks
            Gr-B: Stochastic Differential Equations......................25 marks
AMATH 543  Advanced Paper 2..................................................50 Marks
AMATH 544  Advanced Paper 3..................................................50 Marks
AMATH 545  Gr-A: Integral Equations & Wavelet Transforms......25 Marks
            Gr-B: Project........................................................25 Marks

The course of Total marks 1000 has been divided into 20(twenty) modules
(including Project) each with full marks 50.
LIST OF ADVANCED PAPERS

FOR MODULE AMATH 533 (For 3rd SEMESTER)

AMADV 5301 BASICS OF QUANTUM MECHANICS
AMADV 5302 INCOMPRESSIBLE FLUID MOTION
AMADV 5303 DYNAMICAL MODELS OF ECOLOGY
AMADV 5304 PLASMA KINETIC THEORY
AMADV 5305 ADVANCED OPTIMIZATION AND OPERATIONS RESEARCH -I
AMADV 5306 ADVANCED COMPUTATIONAL METHODS-I
AMADV 5307 THEORY OF ELASTICITY I
AMADV 5308 GEODESY AND GEOPHYSICS-I
AMADV 5309 DYNAMICAL METEOROLOGY
AMADV 5310 DYNAMICAL SYSTEM I
AMADV 5311 GENERAL RELATIVITY
AMADV 5312 ALGORITHMS AND DATA STRUCTURE

FOR MODULES AMATH 543 and AMATH 544 (For 4th SEMESTER)

AMADV 5401 QUANTUM INFORMATION AND QUANTUM COMPUTATION
AMADV 5402 QUANTUM FIELD THEORY
AMADV 5403 COMPRESSIBLE FLOW
AMADV 5404 TURBULENCE
AMADV 5405 COMPUTATIONAL FLUID DYNAMICS
AMADV 5406 DYNAMICS OF COMPLEX ECOLOGICAL SYSTEMS
AMADV 5407 FRACTALS AND CHAOS
AMADV 5408 MATHEMATICAL MODELS IN PHYSIOLOGY AND MEDICINE
AMADV 5409 ECONOPHYSICS
AMADV 5410 FLUID PLASMA THEORY
AMADV 5411 INSTABILITIES AND NONLINEAR PLASMA THEORY
AMADV 5412 ADVANCED OPTIMIZATION -II
AMADV 5413 ADVANCED OPERATIONS RESEARCH -II
AMADV 5414 ADVANCED COMPUTATIONAL METHODS-II
AMADV 5415 ADVANCED COMPUTATIONAL METHODS-III
AMADV 5416 THEORY OF ELASTICITY II
AMADV 5417 THEORY OF ELASTICITY III
AMADV 5418 GEODESY AND GEOPHYSICS-II
AMADV 5419 GEODESY AND GEOPHYSICS-III
AMADV 5420 ATMOSPHERIC THERMODYNAMICS
AMADV 5421 METEOROLOGICAL FORECASTING AND ANALYSIS
AMADV 5422 DYNAMICAL OCEANOGRAPHY
AMADV 5423 DYNAMICAL SYSTEM II
AMADV 5424 DYNAMICAL SYSTEM III
AMADV 5425 GRAVITATION AND BLACK HOLES
AMADV 5426 COSMOLOGY
AMADV 5427 ASTROPHYSICS AND RELATED DATA ANALYSIS
AMADV 5428 THEORY OF AUTOMATA
AMADV 5429 DATABASE MANAGEMENT SYSTEMS; GRAPH THEORY AND COMBINATORICS
AMADV 5430 ASTROSTATISTICS
DETAILED SYLLABUS

Module:-AMATH 411

CLASSICAL MECHANICS AND INTRODUCTORY QUANTUM THEORY

Mechanics of a system of particles: Constraints; Generalized coordinates; Virtual displacement and principle of virtual work; D’Alembert’s principle; Generalized forces; Lagrangian; Lagrange’s equations of motion; cyclic coordinates; velocity dependent potential; Corioli’s force; Principle of energy; Rayleigh’s dissipation function.

Action Integral; Hamilton’s principle; Lagrange’s equations by variational methods; Hamilton’s principle for non-holonomic system; Symmetry properties and conservation laws; Noether’s theorem.

Canonically conjugate coordinates and momenta; Legendre transformation; Hamiltonian; Hamilton’s equations from variational principle; Poincare-Cartan integral invariant; Principle of stationary action; Fermat’s principle; Canonical transformation; Generating function; Poisson Bracket; Equations of motion; Action-angle variables; Hamilton-Jacobi’s equation; Hamilton’s principal function; Hamilton’s characteristics function; Liouville’s theorem.

Planck’s Law, Photo-electric effect; Bohr’s theory, Compton effect; de Broglie waves; Wave-particle dualism; Minimum uncertainty product; Need for a new mechanics; Path Integrals; Fundamental laws and foundations of quantum mechanics; Schrödinger equation; Introductory concepts of quantum states, observables and density matrix formalism.

References:
Module:- AMATH 412

ABSTRACT AND LINEAR ALGEBRA

Group-A:
Abstract Algebra:(30 marks)

Review of general concepts in classical algebraic systems, Dihedral groups, Caley’s theorem. Generator of an arbitrary group, Commutator subgroup, Group automorphisms; Conjugate classes, Centralizer or normalizer; The Class equation, Group Action, Cauchy’s theorem, Sylow’s p-subgroups, Sylow’s theorems and applications; Direct product of groups; Structure theorem for finitely generated Abelian groups.

Group representations, Reducible and irreducible representations, Unitary representation, Schur’s lemma, Orthogonality theorem, Characters of a representation, Continuous groups, Lie groups and their algebras; Generators of a lie group. The linear groups, orthogonal groups and unitary groups with examples, applications.

Basic recollections of rings, integral domains, Fields; Homomorphisms; Ideals and quotient rings; Maximal and prime ideals; Polynomial rings, Factorisation in R[x], Division algorithm, Gauss’ lemma; Irreducibility criteria, Eisenstein’s criterion (statement only). Euclidean domain.
Field extensions; Algebraic extensions; Splitting fields. Finite fields.
The Galois group of a polynomial. The fundamental theorem of Galois theory.

Group-B:
Linear Algebra:(20 marks)

Linear functional on a vector space. Algebraic and geometric multiplicity of eigenvalues; Hermitian and unitary matrices. Orthogonal diagonalizability of Hermitian matrices.
Primary decomposition theorem; Nilpotent matrices; Jordan canonical form.

Normal matrices; Polar and Singular value decomposition.
Invariant subspaces; Generalised eigen vectors and generalised eigen spaces; Generalised Inverse of a matrix.
Linear operators; Properties of Hermitian and Unitary operators; Functions of operators; Eigenvalues of functions of operators; Spectral decomposition.
Bilinear forms; Symmetric, skew symmetric and Hermitian forms; Sylvester’s law of inertia. Spectral theorems for the Hermitian and normal operators on finite dimensional vector spaces.

References:

11. S. Lang, Algebra, Springer-Verlag.
Module:- AMATH 413

REAL ANALYSIS

Group-A: (25 marks)

Derivative Matrix and the Differential; Mean Value Theorem for General Mappings; Chain rule for general mappings; The inverse function and implicit function theorems for general mappings. The related examples.

Bounded and totally bounded metric spaces; Completeness and Baire category theorems; Compact and separable metric spaces. Functions on metric spaces. Limit, continuity and uniform continuity. Contraction mapping. Banach fixed point theorem. Introductory concepts on Banach space, Hilbert space. Simple properties. Wierstrass approximation theorem.


Group-B: (25 marks)


References:

6. M.E. Munroe, Measure and Integration, Addison Wesley, 1953.
15. Donald L. Cohn, Measure Theory, Birkhauser, Boston, 1980.
Module:-AMATH 414

COMPLEX ANALYSIS AND THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

Group-A:
Complex Analysis:(30 marks)

Complex numbers, Topology of the complex plane, sequence of complex numbers, Stereographic projection.
Analytic functions, Cauchy-Riemann equations, Zeros of analytic functions, Multiple valued functions, Branch cuts, Concept of Riemann sheet.
Curves in the complex plane, Complex integration, Jordan's Lemma.
Cauchy's theorem, Morera's theorem, Cauchy integral formula, Maximum modulus principle, Open mapping Theorem, Schwarz Lemma, Liouville's theorem, Fundamental theorem of algebra.
Singularities, Classification of singularities, Cauchy's residue theorem.
Evaluation of some integrals, Argument principle, Rouche's theorem, Hurwitz Theorem.
Conformal mapping, Mobius transformation.

Group-B:
Theory of Ordinary Differential Equations: (20 marks)

Existence and Uniqueness of solutions of initial value problems for first order ordinary differential equations, Singular solutions of first order ODEs.
Linear homogeneous differential equation: Ordinary and singular points, Series solution, Method of Frobenius, Fuch's theorem, Equations of Fuchsian type.
References:

Module:-AMATH 415

PARTIAL DIFFERENTIAL EQUATIONS, GENERALISED FUNCTIONS & INTEGRAL TRANSFORMS

Group-A:
Partial Differential Equations:(25 marks)


Group-B:
Generalized Functions and Integral Transforms:(25 marks)

Generalized Functions:
The Fourier Transform:
The Laplace transform:
The Z-Transform:
Properties of the region convergence of the Z-transform. Inverse Z-transform for discrete-time systems and signals, Signal processing and linear system.
The Hankel transform:
Elementary properties; Inversion theorem; transform of derivatives of functions; Perseval relation; Relation between Fourier and Hankel transform; use of Hankel transform in the solution of PDE.
The Mellin transform:
Definition; properties and evaluation of transforms; Convolution theorem for Mellin transforms; applications to integral equations.
References:

Module:- AMATH 421

CONTINUUM MECHANICS-I AND RIGID DYNAMICS

Group-A:
Continuum Mechanics-I: (25 marks)


Group-B:
Rigid Dynamics: (25 marks)


References:

1. L.E. Malvern, Introduction to the Mechanics of a continuous medium, Prentice-Hall, Inc..
Module:- AMATH 422

CONTINUUM MECHANICS-II

**Inviscid incompressible fluid**: Field equations; Bernoulli’s theorem and applications; Cauchy’s integral; Helmholtz’s equations, Impulsive generation of motion, Kelvin’s circulation theorem of minimum kinetic energy, Irrotational motion in simply connected and multiply connected regions, Three-dimensional motion, image of a source, sink and doublets with respect to a plane and sphere. Translation of sphere in an infinite liquid, D’Alembert’s paradox, Sphere theorem, Axi-symmetrical motion, Stokes’ stream function, Two dimensional motion, Stream function, complex potential, motion of translation and rotation of circular and elliptic cylinders in an infinite liquid, circle theorem, Blasius theorem, Kutta-Joukowski’s theorem.

Constancy of circulation. Permanence of vortex lines and filaments. Equation of surface formed by the streamlines and vortex lines in the case of steady motion, System of vortices, Rectilinear vortices, Vortex pair and doublets, Image of vortex with respect to a circle. A single infinite row of vortices, Karman’s vortex sheet, Pair of stationary vortex filament behind a circular cylinder in a uniform flow. Surface waves, progressive waves in deep and shallow water, Stationary waves, energy and group velocity, waves at the interface of two liquids.

**Viscous incompressible fluid flow**: Field equations, Boundary conditions, Reynold’s number, Vorticity equation, Circulation, Flow through parallel plates, Flow through pipes of circular and elliptic cross sections.


**References:**

4. P.K. Kundu and Iva M.Cohen, Fluid Dynamics, Har Court, India.
5. J.J. Stoker, Water waves, the mathematical theory with application, Interscience Publ.
Module:- AMATH 423

TOPOLOGY, FUNCTIONAL ANALYSIS & OPERATOR THEORY

Topology:


Functional Analysis & Operator Theory:

Linear spaces: Normed linear spaces, Linear topological spaces, Banach spaces, Hilbert spaces, Orthogonality in Hilbert spaces and related theorems (Orthogonal Projection Theorem; Best Approximation; Generalized Fourier Series; Bessel’s Inequality; Complete Orthonormal set; Perseval’s Theorem).


References:

<table>
<thead>
<tr>
<th>No.</th>
<th>Author(s)</th>
<th>Title</th>
<th>Publisher</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>J. Dugundji</td>
<td>Topology</td>
<td>Allyn and Bacon, 1966</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Reprinted by PHI, India)</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>N. Bourbaki</td>
<td>General Topology Part-I</td>
<td>Addison Wesley, Reading</td>
<td>1966</td>
</tr>
<tr>
<td>7.</td>
<td>G. Cain</td>
<td>Introduction to General Topology</td>
<td>Addison Wesley, Reading</td>
<td>1994</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>McGraw Hill, New Delhi</td>
<td>1992</td>
</tr>
</tbody>
</table>
Module:- AMATH 424

OPTIMIZATION TECHNIQUES AND CALCULUS OF VARIATIONS

Group-A:
Optimization:(30 marks)
Linear porgramming, Revised simplex method, Dual simplex method, Post optimal analysis.
Nonlinear programming, Karush-Kuhn-Tucker necessary and sufficient conditions of optimality, Quadratic programming , Wolfe's method, Beale's method.
Dynamic programming, Bellman's principle of optimality, Recursive relations, System with more than one constraint, Solution of LPP using dynamic Programming.
Inter programming, Gomory's cutting plane method , Branch and bound technique. Inventory control, Concept of EOQ, Problem of EOQ with finite rate of replenishment, Problem of EOQ with shortages, Multi-item deterministic problem, Probabilistic inventory models.
Elements of Fuzzy set theory and its relevance in representing uncertainties, Fuzzy linear programming.

Group-B:
Calculus of Variations:(20 marks)
Isoperimetric problem and its solution.
Basic Lemma and fundamental problem in two and three dimensions.

References:
Module: AMATH 425

THEORY OF RELATIVITY AND CLASSICAL ELECTROMAGNETISM

Group-A:
Theory of Relativity: (25 marks)

Galilean Transformations; Inertial frame; Michelson-Morley expriment. Lorentz transformations. Simultaniety, Time-dilation and Lorentz contraction. Minkowski diagram; LT through Minkowski diagram; Constancy of speed of light; Acceleration under LT. Differentiable Manifold; Tangent space; Cotangent space; Scaler product; Transformation rule for covariant and contravariant tensors; Tensor space, properties of tensors; Examples. Metric tensor; properties of metric tensors; Space like and Time like intervals, Relativistic momentum and energy. Affine connection, Co-variant differentiation, Parallel Transport; Geodesic equation; Examples. Principles of General Relativity.

Group-B:
Classical Electromagnetism: (25 marks)


References:

Module:- AMATH 531

DISCRETE MATHEMATICS, GRAPH THEORY AND COMPUTER PROGRAMMING

Group-A: Discrete Mathematics:(20 marks)

Propositional Logic: Propositional Equivalence; Predicates and Quantifiers; Method of Proof; Normal forms; Applications.


Graph Theory:(20 marks)

Diagram of binary relation; Basic definitions; Matrix representations. Graphs and examples, Paths, Cycles, Radius, Diameter, Girth, Circumference; Weighted distance metric; Euler Walk, Hamiltonian cycle. Connectivity, Subgraphs, Isomorphism. Properties of Tree, Rooted binary tree, Spanning trees, Prim’s algorithm. Linear spaces associated with graphs; cycle subspace, Cutset subspace. Relation between bases and spanning trees. Graph colouring, Chromatic number, Gurthrie’s four colour problem and Heawood’s five colour map (statement only). Planarity, Crossing number, Kuratowski’s theorem (statement only). Directed graph, Network flow, Max-flow Min-cut theorem (statement only); Ford and Fulkerson algorithm.

Group-B: Non-Linear Dynamics:(10 Marks)


References:

8. M.M. Morris, Digital logic and computer design, Prentice Hall of India.
13. F. Harary, Graph Theory, Addison-Wesley, 1969.
18. R.J. Wilson, Introduction to Graph Theory, Pearson Education.
19. R. Diestel, Graph Theory, Springer.
20. B. Bollobas, Modern Graph Theory, Springer.


Module: AMATH 532

NUMERICAL ANALYSIS

1. **Computer Number System**: Control of round-off-errors, Instabilities – Inherent and Induced, Hazards in approximate computations, Well posed computations, Well-posed and Ill-posed problems, The direct and inverse and identification problems of computation.


3. **Eigenvalues and Eigenvectors of Real Matrix**: Power method for extreme eigenvalues and related eigenvectors, Jacobi’s method for symmetric matrix (algorithm only), Given’s and Hosueholder’s methods (Algorithms).

4. **Solution of Non-linear Equations**:
   - **Single Equation**: Modified Newton-Raphson method (for real roots-simple or repeated). Aitken’s $\delta^2$-method and Steffensen’s iteration.
   - **Roots of Real Polynomial Equations**: Sensitivity of polynomial roots, Lagrange’s theorem, Hua’s theorem, Bairstow’s method of quadratic factors, Quotient-difference method (algorithm only), Graeffe’s root squaring method.
   - **Non-Linear Systems of Equations**: Newton’s method, Quasi-Newton’s method.

5. **Polynomial Interpolation**: Weirstrass’s approximation theorem (Statement only), Runge’s phenomena, Divergence of sequences of interpolation polynomials for equi-spaced interpolation points, piecewise polynomial interpolation – Hemite interpolation, Error term, Cubic spline interpolation, Convergence properties (statement only), Inverse interpolation, Numerical differentiation using equi-spaced points.

6. **Approximation of Functions**: Least squares polynomial approximation, Approximation with orthogonal polynomials, Chebyshev polynomials, Lanczos economization, Harmonic analysis.

7. **Numerical Integration**: Problem of approximate quadrature, Round-Off errors and Uniform coefficient formulae, Newton-Cotes formulae, Gauss-Legendre and Gauss-chebyshev quadratures, Euler-Maclaurin summation formula, Richardson extrapolation, Romberg integration, Simpson’s adaptive quadrature, Fredholm integral equation, Double integrals – Cubature formula of Simpson Type, Improper integrals.

8. **Numerical Solution of Initial Value Problems for ODE**:

29
**First Order Equation:** Runge-Kutta methods, Multistep predictor-corrector methods – Adams-Bashforth method, Adams-Moulton method, Milne’s method, Convergence and stability.

**Higher Order Equations:** RKF4 method, Stiff differential equations.

**9. Two-point Boundary Value Problems for ODE:** Finite difference methods, Passage method, Shooting method.

**10. Numerical Solution of PDE by Finite Difference Methods:** Parabolic equation in one dimension (Heat equation), Explicit finite difference method, Implicit Crank-Nicolson method, Hyperbolic equation in one-space dimension (Wave Equation) – Finite difference method, Method of characteristics (Consistency, Convergence and Stability).

**References:**

Module:- AMATH 541

NUMERICAL PRACTICAL

Numerical Solution of Polynomial Equations:
Bairstow’s Method, Q-D Algorithm.

Numerical Solution of a System of Equations:
Gauss Elimination Method for a System of Linear Equations (recapitulation),

Computation for Finding Inverse Matrix:
Gauss Elimination Method (recapitulation), L-U Decomposition due to Crout.

Methods for Finding Eigen Pair of a Matrix:
Power Method for Numerically Least Eigen Value and Corresponding Eigen
Vector of a Matrix, Rotation Method of Jacobi for Finding All Eigen Pairs.

Approximation:
Chebychev Approximation, Minimax Approximation.

Harmonic Analysis.

Quadrature Rules:
Gauss Quadrature, Romberg Quadrature.

Numerical Solution of ODE:
Single Step Methods - RKF-45 Scheme, Taylor Series Expansion Method, Some
Starting Methods- Euler’s Formula, Picard’s Formula, Heun’s Formula
(recapitulation), Milne’s Starting Formula, Multistep Methods – Adams-Bashforth
Scheme, Adams-Moulton Scheme, Milne’s Predictor-Corrector Formulae.

Numerical Solution of PDE:
Finite Difference Method for PDE – Elliptic Type PDE, Parabolic Type PDE,
Hyperbolic Type PDE, Crank- Nicholson Scheme for Parabolic Type PDE.

Numerical Solution for BVP:
Passage Method, Linear and Nonlinear Shooting Method.

Numerical Solution of Integral Equation:
Volterra Type IE, Fredholm Type IE.

General Instructions:
Numerical Practical will be based mainly on computer programming. Students at
the end of the semester shall have to submit a report on their sessional works. Use
of different mathematical softwares in numerical computation will be encouraged.
Module:- AMATH 542

STOCHASTIC PROCESS AND STOCHASTIC DIFFERENTIAL EQUATIONS

**Group-A:**
Stochastic Process: (25 marks)

Review of Probability: Sigma field; Probability measure. Random variables; Probability generating functions; Characteristic functions; Probability inequalities; Convergence concepts; random vectors; Bivariate and multivariate distributions.

Conditional Expectation: Conditioning on an event, conditioning on a discrete random variable, conditioning on an arbitrary random variable, conditioning on a sigma-field.

Martingales: Filterations, games of chance, stopping times, optional stopping theorem.


Stochastic process in continuous time: Poisson process and Brownian motion

**Group-B:**
Stochastic Differential Equations: (25 Marks)

Stochastic Calculus; Brownian motion, Wiener Process and white noise, Properties of Brownian paths and Langevin equation, Mean square Calculus, Definition and properties of Ito integral, Indefinite Ito integrals, Ito’s formula, Ito integral in n-dimensions, Simple examples, Concept of Stratonovich integral.

Stochastic differential equations; Definitions and examples; Existence and uniqueness of solutions, Properties of Solutions; Linear stochastic differential equations and its solutions; Kolmogorov backward equation of a stochastic differential equation, Fokker-Planck equation (forward equation) of a stochastic differential equation, meaning of the forward equation, meaning of the backward equation.

**References:**


Module:— AMATH 545

INTEGRAL EQUATIONS & WAVELET TRANSFORMS AND PROJECT

Group-A:
Integral Equations and Wavelet Transforms:(30 marks)

Integral Equations:

Wavelet Transforms:
Definition of wavelet, Examples, Window function, Windowed Fourier transform, Continuous wavelet transform, Discrete wavelet transform, Multiresolution analysis, Application to signal and image processing.

Group-B: Project

References:

ADVANCED PAPERS FOR MODULES AMATH 533, AMATH 543 AND AMATH 544

AMADV 5301

BASICS OF QUANTUM MECHANICS


Schrödinger equation in higher dimensions. Symmetry invariance and angular momentum algebra. Radial equation, Hydrogen atom, O(4) symmetry, Degeneracies.


References:
2. E. Merzbacher, Quantum Mechanics, Wiley, New York, 1970. A. Messiah,
   Quantum Mechanics, North Holland, Amsterdam, 1968.
4. E.J. Squieres, The Mystery of the Quantum World, Adam Hilger, Bristol,
   1986.
5. B.H. Bransden and C.J. Joachain, Introduction to Quantum Mechanics,
   ELBS, 1990.
   Delhi, 2003.
10. J.J. Sakurai, Advanced Quantum Mechanics, Addison-Wesley, ISE Reprint,
    1999.
11. J.J. Sakurai, Modern Quantum Mechanics, 2nd eds., Addison-Wesley, ISE
    Reprint, 1999.
13. C. C. Tannoudji, B. Diu and F. Laloe, Quantum Mechanics, 2 Vols., Wiley-
    2004.
AMADV 5401

QUANTUM INFORMATION AND QUANTUM COMPUTATION


Quantum Information:
Theory of entanglement: Separability and inseparability, bipartite system, pure and mixed entangled states, Schmidt decomposition, Purification, EPR paradox, Hidden variables and Bell inequalities, Separability criteria, Partial transposition and PPT criteria, LOCC.

Distance measures, trace distance, fidelity, Shannon entropy, von-Neumann entropy and is properties (subadditivity, concavity, etc.)


Entanglement measures : Entropy of entanglement, concurrence, relative entropy of entanglement, entanglement of formation and distillable entanglement. Entanglement in multipartite systems.

Quantum computation : Classical and quantum computers, circuit complexity, one- and two-qubit gates, circuit representation, controlled-NOT, universality, geometric gates, DiVincenzo criteria, Deutsch’s algorithm, Grover’s algorithm, Basics of Shor’s algorithm, Quantum error correction, CSS codes, stabilizer code formalism, Clifford codes. Fault-tolerant quantum computation, Implementations of quantum computers, trapped ions, Josephson junctions.

Quantum cryptography: Basics of classical cryptography, Symmetric systems, one time pad, public key systems, RSA, limits of classical cryptography, quantum key
distribution, BB84 and Ekert91 protocols, privacy and coherent information, experiments.

References:

AMADV 5402

QUANTUM FIELD THEORY

Canonical Quantisation: Scalar field and charged scalar field, Radiation field, Indefinite metric and Gupta-Bleuler quantisation, Vacuum fluctuations, Casimir effect, The Dirac field, Gauge fields, Covariant derivatives.

Basic ideas of functional differentiation, Dimensional regularization.

Review of the Path integral formulation of Quantum Mechanics.

Path integral quantisation: Generating functional, Free-particle Green function, Application to $\phi^4$ theory. Connected diagrams.

Quantisation of gauge fields, Photon propagator, Non-Abelian gauge fields and the Faddeev-Popov method, Ghost field, Ward-Takahashi identities.

Renormalisation, Divergence and Renormalisation of $\phi^4$ theory.

Renormalisation group.

References:

INCOMPRESSIBLE FLUID MOTION


References:

AMADV 5403

COMPRESSIBLE FLOW

Field equation of compressible flow
Crocco-Vazsonyl equation
Propagation of small disturbance in a gas
Dynamical similarity of two flows
Plane rotational and irrotational motion with supersonic velocity
Characteristics and their use for solution of plane irrotational problem
Prandtl-Mayer fluid flow past a convex corner
Steady linearized subsonic and supersonic flows. Prandtl-Glauret transformation
Flow along a wavy boundary
Flow past a slight corner
Jangen-Rayleigh method of approximation
Ackeret’s formula Legendre and Molenbrock transformations
Chaplygin’s equation for stream-function
Solution of Chaplygin’s equation
Subsonic gas jet, limiting line
Motion due to a two dimensional source and a vortex
Karman-Tsien approximation
Transonic flow, Euler’s-Tricomi equation
Hyperosnic flow

References:

TURBULENCE

Theory of turbulent mean flow:

Reynold’s Equation
Empirical theorem
Prandtl’s momentum transfer
Karman similarity hypothesis
Flow through a smooth tube
Prandtl’s (1/7)th power law
Boundary layer over a flat plate
Jets and wakes, mixing zone between two parallel streams
Plane jets, axisymmetrical jet
Wake behind a symmetrical cylinder

Statistical Theory of Turbulence:

Correlation function
Taylor’s one-dimensional energy spectrum
The momentum equation
Isotropic turbulence
Properties of the lateral and longitudinal correlation coefficient
Eddy sizes and energy dissipation
Karman-Howarth equation
The energy flux homogeneous isotropic turbulence
The Kolmogorov-Obukhov Law
Effect of finite viscosity: The dissipation range.

References:

AMADV 5405

COMPUTATIONAL FLUID DYNAMICS

Finite difference and finite volume description
Explicit and implicit scheme
Lax-Wendroff scheme for Hyperbolic, elliptic and parabolic equation
Basic concepts of CFD
Introduction to CFD code
Grid generation
Differential equation methods
Algebraic methods
Some simple applications to inviscid incompressible and compressible fluid.

References:

2. A.W. Date. Introduction to Computational Fluid Dynamics (Cambridge).


References:

DYNAMICS OF COMPLEX ECOLOGICAL SYSTEMS


References:

AMADV 5407

FRACTALS AND CHAOS

Fractals: Introduction, Countable and uncountable sets, Cantor set, Fractal properties of Cantor set, Dimension of self-similar fractals, Similarity dimension, More general sets, Box dimension, Pointwise and correlation dimensions, Multifracts, Examples.


References:

MATHEMATICAL MODELS IN PHYSIOLOGY AND MEDICINE


2. **Mathematical models of Heat Physiology**: Basic concepts of heat and heart beat. The local model, the threshold effect. The phase-plane analysis and heart-beat model. Physiological consideration of heart beat cycle. A model of cardiac pacemaker.


References:

Basic Information: Financial Markets

Stock Markets and Derivatives. Three important derivatives – Forward contracts, Futures contracts and options.

Binomial Tree Models


Stochastic Calculus


Continuous Models and Black-Scholes Formula


References:

PLASMA KINETIC THEORY

Short range and long range forces; Some elementary ideas of electric and magnetic fields. Plasma as the fourth state of matter. Thermal ionization, Saha equation, Basic defining properties of plasma, Debye screening, Plasma parameter. Natural occurrence of Plasma. Applications of plasma physics.

Kinetic equations of plasma: Boltzmann equation (Noncollisional and collisional), BBGKY hierarchy equations, Vlasov equations.

Macroscopic Equation of Plasma: Two fluid description of plasma: charge, energy and momentum conservation equations.


Vlasov theory of plasma instability: Two stream instability. Gentle-Bump instability. Penrose criteria of wave plasma instability with application.


References:

AMADV 5410

FLUID PLASMA THEORY

Field of a moving point charge Radiated Power Lorentz Transformation of Electromagnetic field Intensities, Relativistic Motion of Charged Particles, Cerenkov Radiation and Gyroradiation.

Recapitulation of basic plasma properties.

Motion of charged particles in electric and magnetic fields: Larmor orbits, Particle drifts, Adiabatic invariants. Magnetic mirror. Motion of a charge particle in the field of a plane Electromagnetic wave.

Electromagnetic waves in dielectric medium: Propagation characteristics of waves; Dispersion Relation. Poynting Theorem, Wave Energy, Positive and negative energy waves.


References:

AMADV 5411

INSTABILITIES AND NONLINEAR PLASMA THEORY


Waves in Warm Magnetoplasma: Propagation characteristics of Alfvén wave, magneto acoustic wave, CGL model, Firehose instability.


Three wave interaction in plasma. Parametric instability. Stimulated Raman and Brillouin instability in laser produced plasma.

References:


AMADV 5305

ADVANCED OPTIMIZATION AND OPERATIONS RESEARCH -I

Revised Simplex Method:
Product form of Inverse, RSM with and without artificial variables.

Duality:
Complementary slackness conditions, Primal – Dual algorithm.

Post-optimal Analysis and Parametric Programming:
Discrete changes in the cost vector, Discrete changes in the requirement vector, Discrete changes in the coefficient matrix, Addition of a variable, Addition of a constraint, Parameterization of the cost vector and the requirement vector.

Large Scale Linear Programming : Special Computational Techniques:
Composite Simplex Algorithm, Dantzig’s Decomposition Algorithm, Upper bound constraints.

Complexity of the Simplex Algorithm and Polynomial Algorithm:
Polynomial complexity issues, computational complexity of the simplex algorithm, Khachian’s ellipsoid algorithm, Karmarkar’s projective algorithm.

Queueing Theory:
Poisson process and Exponential distribution, Markovian property of exponential distribution, single channel exponential queueing models, steady state solutions for M/M/1 model, Measure of effectiveness, Waiting time distributions, Finite system capacity queues with truncation M/M/1/K, Transient behaviour, Simple Markovian birth-death queueing models, Queues with parallel channels, Queues with parallel channels and truncations.

Stochastic Inventory Models:
Stochastic lot size models and their extensions, probabilistic models, safety and buffer stock, Concept of just-in-time inventory.

Replacement Problems and System Reliability:
Replacement of items that deteriorate with time, Replacement of items that fail completely, Other replacement problem, Basics of reliability, Classes of life distributions, Series, parallel, stand by configuration, Reliability models of maintained and non-maintained system. Renewal theory and its applications, Optimization problem with respect to system reliability.
Geometric Programming:
Polynomial, Geometric-arithmetic mean inequality, Unconstrained GPP, constrained GPP, complementary GP algorithm.

References:

7. Gross and Harris, Queueing Theory, John Wiley.
AMADV 5412
ADVANCED OPTIMIZATION -II

Optimization:
The nature of optimization problem and scope of the theory.

The non-linear programming problem and its fundamental ingredients:
Linear inequalities and theorems of the alternative Farkas theorem. The optimality criteria of linear programming. Thucker’s Lemma and existence theorems, Alternative theorems.

Convex sets-separation theorems:
Convex and Concave functions – Basic properties and some fundamental theorem for convex functions.

Saddle point optimality criteria without differentiability:
The minimization and the local minimization problems and some basic results. Sufficient optimality theorem, Fritz John saddlepoint necessary optimality theorem, Slater’s and Karlinh’s constraint qualifications and their equivalence, The strict constraint qualification, Kuhn-Tucker Saddle point necessary optimality theorems.

Differentiable convex and concave functions:
Some basic properties, twice differentiable convex and concave functions, theorems in cases of strict convexity and concavity of functions.

Optimality Criteria with differentiability:

Duality in non-linear programming:
Weak duality theorem, Wolfe’s duality theorem, strict converse duality theorem, Hanson-Huard strict converse duality theorem, Unbounded dual theorem, Duality in quadratic and linear programming.
Unconstrained Optimization:

Search Methods-Fibonacci search, Golden section search.

Constrained Optimization:

Methods of feasible direction, Cutting hyperplane Method.

References:

Network Analysis:  

Stochastic Programming Problem:  

Optimal Control Theory:  
Performance criterion, Unconstrained systems, Application of calculus of variation, constrained systems, Pontryagin’s principle, Quadratic performance criterion, Regulator problem.

Matrix Game:  

Continuous Games:  

Differentiable Game:  
Two person deterministic continuous differential games, Two person zero-sum differential games, Pursuit games, Co-ordination differential games, Non-cooperative differential game.

Simulation:  
Basic concepts, Monte Carlo method, Random number generation, Waiting the simulation model, New process planning through simulation, Capital budgeting through simulation.
**Information Theory:**
Shannon theory, Measure of information, Entropy – the expected information, Entropy as a measure of uncertainty, Memoryless channel, Conditional entropies, Mutual information, Information process by a channel, Channel capacity, Encoding, Shannon-Fanno encoding procedure.

**References:**

Solutions of Polynomial Equations, Transcendental Equations and System of Equations:

Kizner’s method, Newton-McAuley method, Tessler-Eisenberg method, Brent-Brown method, Laguerre’s method, Muller’s method, Broyden’s method, Computer implementation of the methods.

Interpolation:

Aitken-Neville iterated interpolation scheme, Periodic and non periodic splines, Cubic splines using first derivatives, Cubic splines using second derivative convergence theorem, Application of cubic splines.

Matrix Inversion:


Eigen Value Problems:

Eigen pairs of real non-symmetrical matrices, Computation of eigen values and eigen vectors by (i) Danilevsky method (ii) Krylov method.

Optimization Techniques:

One dimensional minimization methods – Fibonacci method, Golden section method.
Unconstrained optimisation techniques – Direct search methods, Steepest descent method, Quasi-Newton methods, Variable metric method (Daviddon-Fletcher-Powell method), Constrained optimisation techniques – Cutting plane method, Method of feasible directions.

References:

AMADV 5414

ADVANCED COMPUTATIONAL METHODS-II

Approximation:


Spline function approximation to the solution of two point BVP for ODE. B-spline approximation.

Simulation:

Introduction, Methodology of simulation, Generation of random numbers, Monte-Carlo simulation, Simulation of different models, Advantages and limitations of simulation.

Numerical Integration:


Approximate Methods of Solving Integral Equations:

Fredholm integral equation, Volterra integral equation. Spline function approximation to the solution of integral equation.

Fast Fourier Transform:

**Finite Element Method:**
Applications of FEM for ODE and PDE.

**References:**
Numerical Solution of Differential Equations:

General notion of consistency, convergence and stability. The Von Neumann stability test.

Numerical Solution of Boundary Value and Eigenvalue Problems for ODE:

Shooting methods, Multiple shooting methods, Direct methods, Finite difference methods, Error estimates, Order of convergence.

Eigenvalue Problem:

Difference method, Methods based on variational principles.

Computer realization of the methods.

Numerical solution of PDE:


Method of Characteristics:

First order hyperbolic systems, convergence of difference solutions to the exact solution.

Semidiscrete methods for parabolic and hyperbolic PDEs.

Computer realization of the above methods.

Numerical Solution for Stiff differential Equations:

Stiff systems and absolute stability (A-stability), Example and Properties of A-stable methods. Certaine’s method, Jain’s method, V-levels Runge-Kutta methods,

**Ill posed problems:**

Selection method, Quasisolutions, Regularization methods, Regularizing operator, Methods of constructing regularizing operators, Examples of application of the regularization methods. Fredholm integral equation of first kind.

**References:**

AMADV 5307

THEORY OF ELASTICITY I


References:
AMADV 5416

THEORY OF ELASTICITY II


References:

THEORY OF ELASTICITY III


References:

AMADV 5308

GEODESY AND GEOPHYSICS-I

Theory of elastic waves – Body waves and Surface waves

Reflection and Refraction of Seismic Waves. Dispersion of seismic wave energy and amplitude of seismic waves.

Ray Theory: Travel-time analysis.

Seismic evidences on the internal constitution of the earth.

Seismic Instruments: Study and Analysis of Records.

Geophysical Prospecting: Seismic Method.

References:


11. C.B. Officer. Introduction to Theoretical Geophysics. Springer-Verlag, N.Y.


AMADV 5418
GEODESY AND GEOPHYSICS-II

Figure of the Earth: Solid-Liquid Earth, Reference Ellipsoid, Geoid. Dimension of the reference ellipsoid from the measurement of meridian arcs. Geodesic.


Theory of Isostasy.

Geophysical Prospecting : Gravity Method.

Elements of Geomagnetism

Satellite Survey

References:

1. L.L. Nettleton, Geophysical Prospecting for Oil, McGraw Hill, N.Y.

Theory of Continental Drift and Plat Tectonics.

Earthquake Intensity and Magnitude.

Microseism, Tsunami.

Geophysical Prospecting: Magnetic / Electromagnetic Method.

Earthquake Prediction Program. Designing of Earthquake Resistance Structure.

References:


11. C.B. Officer. Introduction to Theoretical Geophysics. Springer-Verlag, N.Y.


DYNAMICAL METEOROLOGY


Barotropic instability. The two-level model.

Sources and sinks of energy. Atmospheric energy balance.

References:

AMADV 5420

ATMOSPHERIC THERMODYNAMICS


Mixing in the atmosphere: Horizontal mixing, vertical mixing, mixing condensation level.


References:

AMADV 5421

METEOROLOGICAL FORECASTING AND ANALYSIS


High impact weather systems. Rossby wave trains. Interaction with Rossby waves.


Turbulent motion. Eddy Viscosity. Sea and Land Breeze temperature.

Severe thunderstorms. Environmental characteristics and mechanisms of initiation. Dry lines, Cold fronts aloft, Multi-cell versus supercell. Storm splitting, Mesoscale convective complex, Squall lines, Mesoscale forecasting.

References:

Basic thermodynamics of the ocean water:
Field equations, equations of conservation of mass, equations of motion, equation of conservation of energy, constitutive equations, equation of evolution of potential vorticity, Boussinesq’s approximation, β-plane approximation.

Wavemotion in the ocean:
Basic equation of linearised waves, Short wavelength waves, gyroscopic waves, internal waves, plane waves in rotating stratified ocean, long waves, long wave equations in a continuously stratified fluid, the pressure equation, the vertical mode for constant stratification, the long gravity waves, the Rossby waves.

Equations of ocean currents:
The equations of mean motion, coefficients of turbulent exchanges, the basic equations in spherical coordinates, quasi-static approximation, the geostrophic motion, the Ekman’s drift currents.

Ekman theory:
The basic equations of Ekman theory, Ekman’s boundary layer, the total flow function and its equations, the western boundary currents, equatorial counter currents.

Two-dimensional model of ocean currents:
General analysis of a two dimensional model, viscous boundary layer, inertial boundary layer, the simple linear model of thermocline.

References:
Dynamical system: flow: existence and uniqueness; local and global solution.
Canonical form of linear system of O.D.E.; hyperbolic flow; contraction; generic properties.
Gronwall’s inequality; continuity w.r.t. initial conditions.
Stability: asymptotic stability; Lyapunov function; Lyapunov’s theorems.
Planar dynamical system; fixed points and their nature; phase space analysis.
Poincare-Bendixon’s theory; Closed orbits. Bendixon’s criterion, Closed orbits
Volterra-Lotka system: Competing species; Poincaré map; connection between
stability of Poincare maps and corresponding flows.
Invariant set; attractor; $\alpha$- and $\omega$-limit cycles; basins of attraction.
Discrete dynamical systems: Linear and nonlinear maps; phase space orbits; fixed
points; periodic and eventually periodic orbits and their stability; Attractors and repellors.
Graphical analysis of orbits of one dimensional maps; logistic map; Tent map;
Baker’s map; Hyperbolicity. Sensitive dependence on initial conditions;
topological transitivity; topological conjugacy and semi-conjugacy of maps.
Chaos; chaotic orbits; graphical illustration using logistic map; Lyapunov
exponent; invariant measure; ergodic map; invariant measure of logistic map.

References:

Nonlinear flows and maps: Stable and unstable manifolds; centre manifold; Hartman-Grobman’s theorem for maps and flows; $C^r$-distance; $C^r$-structural stability (Proofs of theorems excluded).

Differential equation with periodic coefficients: Mathieu’s equation; Floquet theory in $\mathbb{R}^1$ and $\mathbb{R}^2$; stability of periodic solutions.

Implicit function theorem and bifurcation theory: bifurcation of steady solution; saddle-node bifurcation; pitchfork bifurcation; transcritical bifurcation; imperfection theory.

Period-doubling bifurcation: period doubling route to chaos.

Bifurcation of steady solution in $\mathbb{R}^2$: Hopf bifurcation; homoclinic and heteroclinic points; homoclinic bifurcation.

Dynamics of maps: Symbolic dynamics and shift map; properties of shift map; properties of logistic map for parameter value $> 2 + \sqrt{5}$; Cantor set; Cantor set structure of logistic map; topological conjugacy of logistic and shift map; chaotic behaviour of logistic map; Feigenbaum constants.

Sharkowskii’s theorem: Li-Yorke’s theorem for 1D maps.

References:


17. Morse and Feshbach, Methods of Theoretical Physics, Part I, McGraw Hill, Kogaknsha, 1953.
Circle map: rotation number; two dimensional maps; Lyapunov exponents: dynamics of linear maps; hyperbolic toral automorphism: chaos on a torus: Morse-Smale diffeomorphism.

Henon map; standard map; twist map; area preserving maps: invertible maps: dynamics of a bouncing ball on a vibrating plane.


Hamiltonian System: perturbation theory: Kolmoro-v-Arnold-Moser theorem (Statement only) adiabatic invariance of action variable; KAM torus: irrational winding number and KAM stability; breaking of KAM tori.

Two dimensional attractors: solenoid attractor: Henon attractor: attractor of Lorenz type.

Dimension of an attractor: Fractal set: box dimension.

References:


AMADV 5311

GENERAL RELATIVITY

Principles of General Relativity:
Riemannian curvature, Curvature tensor, properties of curvature tensor, Ricci tensor, ricci scalar, Bianchi’s identity.
Symmetry of Riemannian spaces, Killing vector, Homogeneity & isotropy, Curvature of maximally symmetric spaces.
Non-inertial frames, principle of equivalence, principle of general covariance, free particle in curved space time, energy momentum tensor,
Einstein field equations, Schwarzschild exterior solution, Birkoff’s theorem, singularities in Schwarzschild space time, Schwarzschild interior solution.

Applications of General Relativity:
Advance of perihelion of mercury, bending of light, Gravitational red shifts, Basic ideas of gravitational waves.
Schwarzschild solution: deviation and basic features, particle and photon orbits in Schwarzschild metric, Observational tests of general relativity.

Standard theory of cosmology:
Simplifying assumptions of cosmology, the cosmological principle, Expansion of the Universe and redshift, Friedmann-Robertson-Walker models (closed, flat and open Universe), critical density, FRW solutions for simple equations of state of cosmic matter and radiation.
Cosmological constant, de sitter Universe.

Early Universe:
Big bang model, Thermodynamics and thermal history of early universe, baryogenesis, Nucleosynthesis, relic neutrinos and microwave background radiation, Recent observational results.

Inflation:
Problems of the standard cosmological model: Flatness (or fine tuning), Horizon, Monopole
Resolution with an inflated (accelerated expansion): Cosmological Constant: The de Sitter Universe: Scalar fields giving rise to inflation (inflaton) : Problems of inflation – graceful exit etc. Various methods to overcome them: extended inflation, slow roll etc.

Dark Energy:
Type IA supernova as standard candles. Luminosity-Redshift relation – dimming of the
brightness of the supernovae: Resolution: a negative deceleration parameter $q$ – an accelerated expansion: Agents driving acceleration – dark energy. The best candidate – Cosmological constant ($\Lambda$CDM model). Problems with $\Lambda$-discrepancy between the predicted and the observed values: Quintessence models: Qualitative description of the agreement of a presently accelerating universe with other observations – BAO, CMB, Hubble data etc.

References:

5. S. Weinberg, Gravitation and cosmology, Wiley.
AMADV 5425

GRAVITATION AND BLACK HOLES

**Exact solutions of Einstein field equations:**
Minkowski space-time; De Sitter space-time, Robertson-Walker spaces; the solution of the vacuum-field equations, General discussion of the Schwarzschild solution, Isotropic coordinates.

**Black holes and Stellar collapse:**
Characterization of coordinates, Singularities, Space-time diagram in Schwarzschild coordinates, Kruskal coordinates, Event horizons, Black holes – a classical arguments, Observational evidence for black holes; Stellar collapse.

**Cosmological Principle:**
Newtonian cosmology, the red shift, the Einstein universe, the expanding universe, Simplifying assumptions of cosmology, Weyl’s postulate, the Cosmological principle, Observational background, Hubble’s law, Observable parameters in Robertson-Walker models, Cosmological red shift, Apparent brightness, Angular size, Source counts, Olbers Paradox.

**References:**
5. J.V. Narlikar, Lectures on Gravitation and Cosmology.
AMADV 5426

COSMOLOGY

The Friedmann Models:
Einstein field equations simplified for cosmology, Energy tensors of the universe, Random motions in an expanding universe, Dust models, Radiation models, Cosmologies with a non-zero cosmological constant, Space-time singularity, Luminosity distance in different models, Horizons and the Hubble radius; the Particle horizons, the Event Horizon, the Angular size-redshift relation.

Bigbang Cosmology:
The radiation dominated universe, thermodynamics of the early universe, the microwave background, the standard models, early epochs of the universe, cosmological coincidences, the steady-state theory, inflation, the anthropic principle; the horizon problems, flatness problem, entropy problem and the monopole problem in Friedmann cosmology.

Formation of large scale structure in the universe:
The Jeans mass in the expanding universe, the evolution of the Jeans mass, Growth in the post-recombination era, growth in radiation-dominated universes.

References:
ASTROPHYSICS AND RELATED DATA ANALYSIS

Basic Background:
Elementary radiative transfer equations, absorption and emission, atomic processes, continuum and line emission, Telescopes, distance measurements, Hubble’s law.

The Sun:
General features of different regions of sun, sources of solar energy, sun spots and solar cycles, solar wind, solar neutrino puzzle, planets and other objects of solar systems.

Spectral Classifications of Stars:
Saha’s equation, Absolute and apparent magnitudes, Mass luminosity relation, Hertsprung-Russell diagram.

Stellar structure:
Equations for hydrostatic equilibrium, Virial theorem, basic Thermodynamics, polytropic stars, Lane Emden equations and its solutions.

Evolution of stars:
Formation of protostars and stars, Evolution of low and high mass stars, Thermonuclear reactions, Supernova, formation of heavy elements.

Cold compact objects:
Degenerate electron gas, White dwarf, Chandrasekhar limit, Neutron stars, Maximum mass of neutron star, Blackhole.

Binary Stars:
Different types of binary stars, Importance of binary systems, Accretion and gravitational radiation (basic ideas).
Galaxies:

Formation and classification, Density wave theory of the formation of spiral arms, Rotation curves, Missing mass and dark matter, Quasars and active galactic nuclei, Our Galaxy, Oort’s constants, magnetic field.

Elementary data analysis:

Regression problem related to various scaling relations of Astronomical objects. Dimension reduction and clustering problems related to multidimensional data.

Hands on (non credited):

2. Studying the radiation pattern of various antennas in the radio frequency range at 750 MHz.
3. Calculations in General Relativity using symbolic tensor package GRTENSOR.
4. Different multivariate data analytic techniques in astrophysics using Software packages (Skymap Pro-11, TopCat, DS9, sdss dr13 Tutorials, SQL Data retrieval & R-latest version).

REFERENCES:


Data Structure: Introduction, formal definition, implementation of basic data structures – Arrays, Lists, Linked Lists, Stacks, Queues, Dequeue, Priority Queue, Recursions, Trees and Graphs.

Sorting and Searching: Sorting techniques with complexity analyses – Insertion, Selection, Merging, Radix Sort, Quick Sort, Heap Sort, etc. Sequential search, Binary search, Indexed sequential search.

Trees: Binary trees, Traversals of binary trees, Threaded binary trees, Binary search trees, AVL tree, B-trees.

Hashing: Basic ingredients, Analysis of hashing with chaining and with open addressing.

References:

Sequential Machine without Output:
Introduction, response functions, accessible states and connected machines, the free automation, congruence relations and homomorphisms. Quotient machines.

Sequential Machines with Output:
The behaviour of sequential machines, behavioural equivalence, Mealy machine, Moore machine.

The Minimization Problem:
The minimal machine having a given behaviour, relations on the set of states, algorithm for computing the minimal machine.

Finite Automata:

Transition system:
Transition systems and regular expressions, the subset construction, the language of regular expressions, the analysis and synthesis theorems, applications of the analysis and synthesis theorems.

Hardware Realization and the State-assignment Problem:
Hardware realization, decomposition of machines.

Probabilistic Machines:
Probabilistic machines, the behaviour of probabilistic machines.

Grammar and Languages:

Turing Machine:
Basic definitions. Turing machine as language acceptors. Universal Turing machine.

References:


4. L. Taylor, Booth, Sequential Machines and Automata Theory (John Wiley and Sons).


8. Hopcroft and Ullman, Formal Languages and Their Relation to Automata (Addison-Wesley).

9. J.E. Hopcroft and J.D. Ulman, Introduction to Automata Theory, Languages and computation (Addison-Wesley).


Database Management System: Overview of file organization technique: Sequential, direct, indexed, hashed, inverted, B-Trees.

Data Models: Relational, Network, Hierarchical.

Relational Model – Algebra, Calculus, Normal forms, Implementation of query languages, Security and protection of data recovery methods.

Concurrent Operations on Databases: Introduction to distributed database system. Case Studies.

Graph Theory and Combinatorics:


References:


8. J.E. Hopcroft and J.D. Ullman, Introduction to automata theory, languages and computation (Addison-Wesley).


Basic Background: Elementary radiative transfer equations, absorption and emission, atomic processes. Distance measurement in Astronomy. Hubble’s law


Evolution of Stars: Observational basis, Sources of stellar energy, Hertzsprung-Russell diagram, evolution of low and high mass stars, Chandrasekhar limit, Pulsars, neutron stars and black holes.

Stellar Populations- Galactic and Globular Clusters, Initial Mass Function (IMF)

Galaxies: Our Galaxy, External galaxies, Clusters of galaxies.

Application of Linear and Nonlinear Regression to various scaling relations for astrophysical objects e.g., Fundamental plane or relations among rotational measures for globular clusters.

Application of Stepwise Regression for extracting significant parameters in many parameter case in Astrophysical situation.

Study of Initial Stellar Mass Functions (IMF) through Monte Carlo Simulation.

Study of variable stars through Time Series Analysis.

Application of Principal Component Analysis.

Classification and Clustering, Discriminant Analysis, Factor Analysis for galaxies and globular clusters.
Hands on:
Applications of Astrophysical Problems through R, sdss, Aladin, Vizier.

References:


ANNEXURE- I

Choice Based Credit Course A/B : 50 Marks

Name: Some Methods in Applied Mathematics: (30 + 20)

Gr-A : Differential equations: (30 Marks = 20 + 10)

(i) Ordinary Differential Equation: (20)

Existence and Uniqueness of solutions of initial value problems for first order ordinary differential equations, Singular solutions of first order ODEs.


(ii) Partial Differential Equation: (10)


Gr-B : Numerical Analysis: (20 Marks)

Computer Number System: Control of round-off-errors, Instabilities – Inherent and Induced, Hazards in approximate computations, Well posed computations, Well-posed and Ill-posed problems, The direct and inverse and identification problems of computation.


References: